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Tensor charges and form factors of SU(3) baryons in the self-consistent SU(3) chiral quark-soliton model

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Abstract

We investigate the tensor form factors of the baryon octet within the framework of the chiral quark-soliton model, emphasizing those of the nucleon, taking linear $1/N_c$ rotational as well as linear m_s corrections into account, and applying the symmetry-conserving quantization. We explicitly calculate the tensor form factors $H_T^q(Q^2)$ corresponding to the generalized parton distributions $H_T(x, \xi, t)$. The tensor form factors are obtained for the momentum transfer up to $Q^2 \leq 1\,\mathrm{GeV}^2$ and at a renormalization scale of $0.36\,\mathrm{GeV}^2$. We find for the tensor charges $\delta u = 1.08$, $\delta d = -0.32$, $\delta s = -0.01$ and discuss their physical consequences, comparing them with those from other models. Results for tensor charges for the baryon octet are also given.

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I. INTRODUCTION

At leading twist, the basic quark structure of the nucleon is described by three independent quark parton distribution functions (PDFs): The unpolarized PDF $q(x)$, the helicity PDF $\Delta q(x)$, and transversity PDF $\delta q(x)$ for each flavor. More generally, eight generalized parton distributions (GPDs) contain full information on the structure of the nucleon: Two chiral-even spin-independent GPDs $H(x,\xi,t)$ and $E(x,\xi,t)$, two chiral-even spin-dependent GPDs $\tilde{H}(x,\xi,t)$ and $\tilde{E}(x,\xi,t)$, and four chiral-odd spin-dependent GPDs $H_T(x,\xi,t)$, $E_T(x,\xi,t)$,$\tilde{H}_T(x,\xi,t)$, and $\tilde{E}_T(x,\xi,t)$[1–5]. Thus, knowing all these leading-twist GPDs will provide a detailed description of the nucleon structure, so called a *nucleon tomography*.

The helicity distributions are related to the axial-vector current of the nucleon, which are relatively easily accessible in experiment because of their chiral-even character. On the contrary, the chiral-odd distributions that are pertinent to the tensor current of the nucleon are rather difficult to be measured. Since quantum chromodynamics (QCD) possesses an approximate chiral symmetry and all electroweak vertices preserve chirality, properties of the tensor current are experimentally very hard to be accessed. There exist no probes to measure directly the tensor structure, so that one is restricted to scattering reactions in which two chiral-odd processes are involved. For example, the transverse spin asymmetry A_{TT} in Drell-Yan processes in $p\overline{p}$ reactions [6–9] as well as the azimuthal single spin asymmetry in semi-inclusive deep inelastic scattering (SIDIS) [10] are seen as promising reactions to gain information on the transversity of the nucleon.

Because of this difficulty, an experimental extraction of the transversity distribution $\delta q(x)$ was only recently performed for the first time [10]. Based on the azimuthal single spin asymmetry in SIDIS processes $lp^{\uparrow} \to l\pi X$, Ref. [10] used experimental data from the Belle Collaboration at KEK [11] as well as data from the HERMES [12, 13] and COMPASS [14] collaborations in order to obtain results for the u and d quark transversity distributions $\delta u(x)$ and $\delta d(x)$. Subsequently, Ref. [15] extracted the tensor charges $\delta u = 0.46^{+0.36}_{-0.28}$ and $\delta d = -0.19^{+0.30}_{-0.23}$ at a renormalization scale of $\mu^2 = 0.4\,\mathrm{GeV}^2$. Anselmino et al. [16] presented an updated analysis and obtained the results $\delta u = 0.54^{+0.09}_{-0.22}$ and $\delta d = -0.23^{+0.09}_{-0.16}$ at a scale of $\mu^2 = 0.8\,\mathrm{GeV}^2$. Moreover, several theoretical investigations on the tensor charges were carried out, for example, in the non-relativistic quark model, in the MIT bag model [17], in SU(6)-symmetric quark models [18, 19], in a valence quark model with axial-vector meson dominance [20], and on the lattice [21].

In the present work, we will study the tensor form factors up to a momentum transfer of $Q^2 \leq 1\,\mathrm{GeV}^2$ within the framework of the self-consistent SU(3) chiral quark-soliton model (χQSM) [22]. The χQSM provides an effective chiral model for QCD in the low-energy regime with constituent quarks and the pseudo-scalar mesons as the relevant degrees of freedom. Moreover, this model has a deep connection to the QCD vacuum based on instantons [23] and has only a few free parameters. These parameters can mostly be fixed to the meson masses and meson decay constants in the mesonic sector. The only free parameter is the constituent quark mass that is also fixed by reproducing the proton electric form factor. We obtain numerically an explicit (self-consistent) pion-field, i.e. the non-topological soliton field, which can be used to calculate basically all baryon observables. A merit of this model is that we are able to determine baryon form factors of various currents within exactly the same relativistic framework and with the same set of parameters. Furthermore, the renormalization scale for the χQSM is naturally given by the cut-off parameter for the

regularization which is about $0.36\,\mathrm{GeV}^2$. Note that it is implicitly related to the inverse of the size of instantons ($\overline{\rho} \approx 0.35\,\mathrm{fm}$) [24, 25]. In particular, this renormalization scale is of great importance in investigating the tensor charges of the proton. In contrast to the axial-vector charges the tensor charges depend on the renormalization scale already at one-loop level.

The axial-vector form factors were calculated in Refs. [26–29] with the same parameters as used in the present work. The tensor charges, i.e. the tensor form factors at $Q^2 = 0$, were already investigated in the SU(2) version of the χQSM in Refs. [30, 31] and in the SU(3) version in Ref. [32]. However, the SU(3) calculation [32] was done prior to the finding of the symmetry-conserving quantization [33] which ensures the correct electromagnetic gauge invariance. Reference [34] formulated the χQSM on the light-cone, within which the tensor charges were studied in [35]. In the case of the SU(2) χQSM the tensor charges were also calculated from the first moments of the transversity distribution $\delta q(x)$ in the references [36–38]. In the present work, we will extend the previous work [32] and calculate the tensor form factors up to $Q^2 = 1\,\mathrm{GeV}^2$ with application of the symmetry-conserving quantization.

The outline of this work is sketched as follows. In Section II we recapitulate the form factors of the tensor current such that it can be used in the χQSM. In Section III, we show how to compute the tensor form factors of SU(3) baryons within the χQSM. Section IV presents the main results of this work and discusses them in comparison with those of other works. The last Section is devoted to a summary and conclusion. The explicit expressions for the quark densities are given in the Appendix.

II. GENERAL FORMALISM

The matrix element of the quark tensor current between two nucleon states is parametrized by three form factors [5, 21, 39] as follows

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^{\chi}\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[H_T^{\chi}(Q^2)i\sigma^{\mu\nu} + E_T^{\chi}(Q^2)\frac{\gamma^{\mu}q^{\nu} - q^{\mu}\gamma^{\nu}}{2M} + \tilde{H}_T^{\chi}(Q^2)\frac{(n^{\mu}q^{\nu} - q^{\mu}n^{\nu})}{2M^2}\right]u_s(p)\,, \tag{1}$$

where $\sigma_{\mu\nu}$ is the spin operator $i[\gamma_\mu,\,\gamma_\nu]/2$ and λ^{χ} the Gell-Mann matrices where we include the unit matrix $\lambda^0 = 1$. The ψ represents the quark field. The $u_s(p)$ denotes the spinor for the nucleon with mass M, momentum p and the third component of its spin s. The momentum transfer q and the total momentum are defined respectively as $q = p' - p$ with $q^2 = t = -Q^2$ and $n = p' + p$. In the language of GPDs the above-defined form factors are equivalent to the following expressions[1]:

$$\int_{-1}^{1} dx\, H_T^{\chi}(x,\xi,t) = H_T^{\chi}(q^2),\ \int_{-1}^{1} dx\, E_T^{\chi}(x,\xi,t) = E_T^{\chi}(q^2),\ \int_{-1}^{1} dx\, \tilde{H}_T^{\chi}(x,\xi,t) = \tilde{H}_T^{\chi}(q^2). \tag{2}$$

In the present work, we will concentrate on the tensor form factors $H_T(Q^2)$ that can be related to the spatial part of the above matrix element in the Breit frame

$$\varepsilon^{nkl}\langle N'_{s'}|\overline{\psi}(0)i\sigma_{kl}\lambda^{\chi}\psi(0)|N_s\rangle = H_T^{\chi}(Q^2)i2\phi_{s'}\left[\sigma^n + q^n\frac{\boldsymbol{q}\cdot\boldsymbol{\sigma}}{4M(E+M)}\right]\phi_s \tag{3}$$

[1] In the notation of the generalized form factors of [40] the above form factors are equivalent to $H_T^{\chi}(q^2) = A_{T10}^{\chi}(t)$, $E_T^{\chi}(q^2) = B_{T10}^{\chi}(t)$ and $\tilde{H}_T^{\chi}(q^2) = \tilde{A}_{T10}^{\chi}(t)$.

with $E = \sqrt{M^2 + \boldsymbol{p}^2}$ as the nucleon energy, ϕ_s the two-component spinor and $N' = N(p')$. In order to derive the expressions for $H_T^\chi(Q^2)$, we take the third component of the space, i.e. $n = 3$ and perform an average over the orientation of the momentum transfer. Then we act first $\int \frac{d\Omega}{4\pi}[\boldsymbol{q} \times (\boldsymbol{q} \times$ on both sides, and take the average. The results are found to be

$$\int \frac{d\Omega_q}{4\pi} \langle N'_{\frac{1}{2}} | T_z^\chi | N_{\frac{1}{2}} \rangle \;=\; H_T^\chi(Q^2)\, i \frac{2M + E}{M} \frac{2}{3} - E_T^\chi(Q^2)\, i \frac{|\boldsymbol{q}|^2}{M^2} \frac{1}{3}\,, \tag{4}$$

$$\int \frac{d\Omega_q}{4\pi} \left[\boldsymbol{q} \times \left(\boldsymbol{q} \times \langle N'_{\frac{1}{2}} | \boldsymbol{T} | N_{\frac{1}{2}} \rangle\right)\right]_z \;=\; -H_T^\chi(Q^2)\, i |\boldsymbol{q}|^2 \frac{4}{3} + E_T^\chi(Q^2) \frac{i}{M^2} |\boldsymbol{q}|^4 \frac{1}{3} \tag{5}$$

with $\boldsymbol{T}^\chi = i\varepsilon^{nkl} \overline{\psi}(0) \sigma_{kl} \lambda^\chi \psi(0) \hat{\boldsymbol{e}}_n$. We can therefore separate H_T^χ and E_T^χ as

$$H_T^\chi(Q^2) \;=\; 3\frac{M}{E} \int \frac{d\Omega_q}{4\pi} \left\{ \boldsymbol{T}_{NN}^\chi + \frac{1}{|\boldsymbol{q}|^2} \left[\boldsymbol{q} \times (\boldsymbol{q} \times \boldsymbol{T}_{NN}^\chi)\right] \right\}_z\,, \tag{6}$$

$$E_T^\chi(Q^2) \;=\; \frac{12M^3}{E|\boldsymbol{q}|^2} \int \frac{d\Omega_q}{4\pi} \left\{ \boldsymbol{T}_{NN}^\chi + \frac{2M + E}{2M|\boldsymbol{q}|^2} \left[\boldsymbol{q} \times (\boldsymbol{q} \times \boldsymbol{T}_{NN}^\chi)\right] \right\}_z\,, \tag{7}$$

using the relation $i\varepsilon^{nkl}\sigma_{kl} = 2i\gamma^0\gamma^n\gamma^5$ and the definition

$$\boldsymbol{T}_{NN}^\chi \;:=\; \langle N'_{\frac{1}{2}} | \psi^\dagger(0) \boldsymbol{\gamma} \gamma^5 \lambda^\chi \psi(0) | N_{\frac{1}{2}} \rangle\,. \tag{8}$$

Equations (6,7) can be now evaluated in the χQSM. We want to note that both form factors involve the expressions $(\boldsymbol{T}_{NN}^\chi)_z$ and $[\boldsymbol{q} \times (\boldsymbol{q} \times \boldsymbol{T}_{NN}^\chi)]_z$. However, we will concentrate in the present work on the tensor form factor $H_T^\chi(Q^2)$. Even though both form factors consist of the same two densities, the second form factor $E_T(Q^2)$ requires more technical efforts for the region of small Q^2. In addition, the third form factor, $\tilde{H}_T^\chi(Q^2)$ requires a completely new density. These two form factors will be discussed in a future work. At this point we also see explicitly the difference of the tensor form factors from the axial-vector ones for which the nucleon matrix element is given as

$$\boldsymbol{A}_{NN}^\chi \;=\; \langle N'_{\frac{1}{2}} | \psi^\dagger(0) \gamma^0 \boldsymbol{\gamma} \gamma^5 \lambda^\chi \psi(0) | N_{\frac{1}{2}} \rangle\,. \tag{9}$$

The $\boldsymbol{A}_{NN}$ is distinguished from $\boldsymbol{T}_{NN}$ by only a factor of γ^0. It indicates that the tensor current turns out to be anti-Hermitian whereas the axial-vector one is Hermitian. However, in the nonrelativistic limit the tensor form factors coincide with the axial-vector ones, because γ^0 is replaced by the unity matrix in this limit.

In the following we will give additional expressions which will be used later in the present work. With the above defined current we have the following relations between the individual flavor decompositions in SU(3) as:

$$H_T^0(0) = g_T^0 \;=\; \delta u + \delta d + \delta s \tag{10}$$

$$H_T^3(0) = g_T^3 \;=\; \delta u - \delta d \tag{11}$$

$$H_T^8(0) = g_T^8 \;=\; \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s), \tag{12}$$

and in SU(2) with $\tau^0 = 1$:

$$H_T^0(0) = g_T^0 \;=\; \delta u + \delta d \tag{13}$$

$$H_T^3(0) = g_T^3 \;=\; \delta u - \delta d. \tag{14}$$

We want to note that in the literature several notations for the SU(3) singlet g_T^0 and non-singlet g_T^8 quantities exist. These are due to the fact that either $\lambda^0 = \sqrt{2/3}\cdot 1$ are chosen or the factor $\sqrt{1/3}$ is taken out from g_T^8.

Additionionally, in order to compare the present results of the tensor charges with those of other works, it is essential to know the renormalization scale. Different values obtained at different scales can be connected by following NLO evolution equation [41, 42]:

$$\delta q(\mu^2) \;=\; \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)}\right)^{4/27}\left[1-\frac{337}{486\pi}\left(\alpha_S(\mu_i^2)-\alpha_S(\mu^2)\right)\right]\delta q(\mu_i^2)\,, \tag{15}$$

$$\alpha_S^{NLO}(\mu^2) \;=\; \frac{4\pi}{9\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\left[1-\frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}{\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\right] \tag{16}$$

with $\Lambda_{\mathrm{QCD}} = 0.248$ GeV, the initial renormalization scale μ_i^2 and $N_c = N_f = 3$.

III. SU(3) CHIRAL QUARK-SOLITON MODEL

We will now briefly describe the SU(3) χQSM. We follow the notation used in Refs. [43–46] and for details we refer to Refs. [22, 47, 48]. The SU(3) χQSM is characterized by the following low-energy effective partition function in Euclidean space

$$\mathcal{Z}_{\chi\mathrm{QSM}} \;=\; \int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U\exp\left[-\int d^4x\Psi^\dagger iD(U)\Psi\right] = \int\mathcal{D}U\exp(-S_{\mathrm{eff}}[U])\,, \tag{17}$$

where ψ and U represent the quark and pseudo-Goldstone boson fields, respectively. The S_{eff} denotes the effective chiral action

$$S_{\mathrm{eff}}(U) \;=\; -N_c\mathrm{Tr}\ln iD(U)\,, \tag{18}$$

where Tr designates the functional trace, N_c the number of colors, and $D(U)$ the Dirac differential operator

$$D(U) \;=\; \gamma_4(i\partial\!\!\!/ - \hat{m} - MU^{\gamma_5}) = -i\partial_4 + h(U) - \delta m \tag{19}$$

with

$$\delta m \;=\; \frac{-\overline{m}+m_s}{3}\gamma_4\mathbf{1} + \frac{\overline{m}-m_s}{\sqrt{3}}\gamma_4\lambda^8 = M_1\gamma_4\mathbf{1} + M_8\gamma_4\lambda^8\,. \tag{20}$$

In the present work we assume isospin symmetry, so that the current quark mass matrix is defined as $\hat{m} = \mathrm{diag}(\overline{m},\,\overline{m},\,m_{\mathrm{s}}) = \overline{m} + \delta m$. The SU(3) single-quark Hamiltonian $h(U)$ is given by

$$h(U) \;=\; i\gamma_4\gamma_i\partial_i - \gamma_4 MU^{\gamma_5} - \gamma_4\overline{m}\,, \tag{21}$$

where U^{γ_5} represents the chiral field for which we assume Witten's embedding of the SU(2) soliton into SU(3)

$$U^{\gamma_5}(x) \;=\; \begin{pmatrix} U_{\mathrm{SU(2)}}^{\gamma_5}(x) & 0 \\ 0 & 1 \end{pmatrix} \tag{22}$$

with the SU(2) pion field $\pi^i(x)$ as

$$U^{\gamma_5}_{SU(2)} \;=\; \exp(i\gamma^5\tau^i\pi^i(x)) \;=\; \frac{1+\gamma^5}{2}U_{SU(2)} + \frac{1-\gamma^5}{2}U^\dagger_{SU(2)}\,. \tag{23}$$

The integration over the pion field U in Eq. (17) can be carried out by the saddle-point approximation in the large N_c limit due to the N_c factor in Eq. (18). The SU(2) pion field U is expressed as the most symmetric hedgehog form

$$U_{\rm SU2} \;=\; \exp[i\gamma_5\hat{n}\cdot\boldsymbol{\tau}P(r)]\,, \tag{24}$$

where $P(r)$ is the radial profile function of the soliton.

The baryon state $|B\rangle$ in Eq. (8) is defined as an Ioffe-type current consisting of N_c valence quarks in the χQSM:

$$|B(p)\rangle \;=\; \lim_{x_4\to-\infty}\frac{1}{\sqrt{\mathcal{Z}}}\,e^{ip_4x_4}\int d^3\boldsymbol{x}\,e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\,J^\dagger_B(x)\,|0\rangle \tag{25}$$

with

$$J_B(x) \;=\; \frac{1}{N_c!}\,\Gamma_B^{b_1\cdots b_{N_c}}\,\varepsilon^{\beta_1\cdots\beta_{N_c}}\,\psi_{\beta_1 b_1}(x)\cdots\psi_{\beta_{N_c}b_{N_c}}(x)\;\;, \tag{26}$$

where the matrix $\Gamma_B^{b_1\ldots b_{N_c}}$ carries the hyper-charge Y, isospin I, I_3 and spin J, J_3 quantum numbers of the baryon and the b_i and β_i denote the spin-flavor- and color-indices, respectively. Having minimized the action in Eq. (18), we obtain an equation of motion which is numerically solved in a self-consistent manner with respect to the function $P(r)$ in Eq. (24). The corresponding unique solution U_c is called the classical chiral soliton.

So far, we did not introduce any quantum numbers for the system. This is done by quantizing the rotational and translational zero-modes of the soliton. The rotations and translations of the soliton are implemented by

$$U(\boldsymbol{x},t) = A(t)U_c(\boldsymbol{x}-\boldsymbol{z}(t))A^\dagger(t)\,, \tag{27}$$

where $A(t)$ denotes a time-dependent SU(3) matrix and $\boldsymbol{z}(t)$ stands for the time-dependent translation of the center of mass of the soliton in coordinate space. The rotational velocity of the soliton $\Omega(t)$ is now defined as

$$\Omega = \frac{1}{i}A^\dagger\dot{A} = \frac{1}{2i}{\rm Tr}(A^\dagger\dot{A}\lambda^\alpha)\lambda^\alpha = \frac{1}{2}\Omega_\alpha\lambda^\alpha. \tag{28}$$

Thus, treating $\Omega(t)$ and δm perturbatively with a slowly rotating soliton assumed and with δm regarded as a small parameter, we derive the collective Hamiltonian of the χQSM [49] expressed as

$$H_{coll} \;=\; H_{\rm sym} + H_{\rm sb}\;\;, \tag{29}$$

$$H_{\rm sym} \;=\; M_c + \frac{1}{2I_1}\sum_{i=1}^{3}J_iJ_i + \frac{1}{2I_2}\sum_{a=4}^{7}J_aJ_a, \tag{30}$$

$$H_{\rm sb} \;=\; \frac{1}{m}M_1\Sigma_{SU(2)} + \alpha D^{(8)}_{88}(A) + \beta Y + \frac{\gamma}{\sqrt{3}}D^{(8)}_{8i}(A)J_i\,. \tag{31}$$

Diagonalizing the collective Hamiltonian we derive the octet baryon states

$$|N_8\rangle \;=\; |8_{1/2},N\rangle + c_{\overline{10}}\sqrt{5}|\overline{10}_{1/2},N\rangle + c_{27}\sqrt{6}|27_{1/2},N\rangle, \tag{32}$$

where $c_{\overline{10}}$ and c_{27} are mixing parameters expressed as

$$c_{\overline{10}} \;=\; -\frac{I_2}{15}\left(\alpha+\frac{1}{2}\gamma\right), \quad c_{27} \;=\; -\frac{I_2}{25}\left(\alpha-\frac{1}{6}\gamma\right), \tag{33}$$

and α and γ represent the effects of SU(3) symmetry breaking written as

$$\alpha \;=\; \frac{1}{\overline{m}}\frac{1}{\sqrt{3}}M_8\Sigma_{SU(2)} - \frac{N_c}{\sqrt{3}}M_8\frac{K_2}{I_2}, \quad \gamma \;=\; -2\sqrt{3}M_8\left(\frac{K_1}{I_1}-\frac{K_2}{I_2}\right). \tag{34}$$

The moments of inertia I_1,I_2 and K_1, K_2 can be found, for example, in Ref. [43]. In the χQSM the constituent quark mass M of Eq. (21) is in general momentum-dependent and introduces a natural regularization scheme for the divergent quark loops in the model. However, it is rather difficult to treat the momentum-dependent constituent quark mass within the present model. Instead, we will take it as a free and constant parameter, and introduce a regularization scheme such as the proper-time regularization. It is well known that the value of $M = 420\,\mathrm{MeV}$ together with the proper time regularization reproduces very well experimental form factor data for the SU(3) baryons [22, 26, 27, 47, 48]. We want to mention that in the calculation of nucleon structure function the Pauli-Villars regularization is usually employed [50]. A detailed formalism for the zero-mode quantization can be found in Refs. [22, 48, 49].

IV. TENSOR FORM FACTORS IN THE CHIRAL QUARK-SOLITON MODEL

In this Section we give the final expressions for the tensor form factor $H_T^\chi(Q^2)$ of Eq. (6) evaluated in the χQSM. In the present framework, linear-order corrections coming from $\Omega(t)$ and δm are taken into account while the translation of the soliton is treated only to the zeroth order. Keeping the notations of Refs. [43, 44, 46], we find that Eq. (6) turns out to be

$$H_T^\chi(Q^2) \;=\; \frac{M}{E}\int dr\, r^2\left[j_0(|\boldsymbol{Q}|r)\,\mathcal{H}_{T0}^\chi(r) \;+\; \sqrt{2}j_2(|\boldsymbol{Q}|r)\,\mathcal{H}_{T2}^\chi(r)\right], \tag{35}$$

where the indices χ denote the singlet ($\chi = 0$) and nonsinglet ($\chi = 3, 8$) parts of the tensor form factors. The $j_0(|\boldsymbol{Q}|r)$ and $j_2(|\boldsymbol{Q}|r)$ stand for the spherical Bessel functions. The nucleon matrix element $\mathcal{H}_{T0}^\chi(r)$ is given explicitly in SU(3) as follows:

$$\begin{aligned}\mathcal{H}_{T0}^{\chi=3,8}(r) \;=\; & -\sqrt{\frac{1}{3}}\langle D_{\chi 3}^{(8)}\rangle_N\mathcal{A}_{T0}(r) \;+\; \frac{1}{3\sqrt{3}}\frac{1}{I_1}\langle D_{\chi 8}^{(8)}J_3\rangle_N\mathcal{B}_{T0}(r)\\ & -\sqrt{\frac{1}{3}}\frac{1}{I_2}\langle d_{ab3}D_{\chi a}^{(8)}J_b\rangle_N\mathcal{C}_{T0}(r) \;-\; \frac{1}{3\sqrt{2}}\frac{1}{I_1}\langle D_{\chi 3}^{(8)}\rangle_N\mathcal{D}_{T0}(r)\\ & -\frac{2}{3\sqrt{3}}\frac{K_1}{I_1}M_8\langle D_{83}^{(8)}D_{\chi 8}^{(8)}\rangle_N\mathcal{B}_{T0}(r) + \frac{2}{\sqrt{3}}\frac{K_2}{I_2}M_8\langle d_{ab3}D_{8a}^{(8)}D_{\chi b}^{(8)}\rangle_N\mathcal{C}_{T0}(r)\\ & -\sqrt{\frac{1}{3}}\left[2M_1\langle D_{\chi 3}^{(8)}\rangle_N + \frac{2}{\sqrt{3}}M_8\langle D_{88}^{(8)}D_{\chi 3}^{(8)}\rangle_N\right]\mathcal{H}_{T0}(r)\\ & +\frac{2}{3\sqrt{3}}M_8\langle D_{83}^{(8)}D_{\chi 8}^{(8)}\rangle_N\mathcal{I}_{T0}(r) - \frac{2}{\sqrt{3}}M_8\langle d_{ab3}D_{\chi a}^{(8)}D_{8b}^{(8)}\rangle_N\mathcal{J}_{T0}(r)\,,\end{aligned} \tag{36}$$

$$\mathcal{H}_{T0}^0(r) \;=\; \frac{1}{3}\frac{1}{I_1}\langle J_3\rangle_N\mathcal{B}_{T0}(r) - \frac{2}{3}\frac{K_1}{I_1}M_8\langle D_{83}^{(8)}\rangle_N\mathcal{B}_{T0}(r) + \frac{2}{3}M_8\langle D_{83}^{(8)}\rangle_N\mathcal{I}_{T0}(r)\,, \tag{37}$$

where $\mathcal{A}_{T0}, \cdots, \mathcal{J}_{T0}$ are the quark densities found in the Appendix. The terms with M_1 or M_8 are strange-quark mass (m_s) corrections arising from the operator. The operator J_3 is the third component of the spin operator. The $D^{(8)}$ represent the SU(3) Wigner functions and $\langle\rangle_N$ are their matrix elements sandwiched between the collective nucleon wave functions given in Eq. (32). The results of these matrix elements are finally given in terms of the SU(3) Clebsch-Gordan coefficients.

From the above SU(3) expressions, Eqs. (36,37), we can deduce straightforwardly the corresponding expressions for the SU(2) version. Since there is no strange quark in SU(2), most of the above terms are not present and Eqs. (36,37) are reduced in SU(2) to the following isovector and isosinglet expressions:

$$\mathcal{H}^3_{T0}(r) = -\sqrt{\frac{1}{3}}\langle D_{33}\rangle_N \mathcal{A}_{T0}(r) - \frac{1}{3\sqrt{2}}\frac{1}{I_1}\langle D_{33}\rangle_N \mathcal{D}_{T0}(r) \tag{38}$$

$$\mathcal{H}^0_{T0}(r) = \frac{1}{3}\frac{1}{I_1}\langle J_3\rangle_N \mathcal{B}_{T0}(r), \tag{39}$$

with $\langle D_{33}\rangle_N = -1/3$ and $\langle J_3\rangle_N = 1/2$ as the corresponding SU(2) matrix elements.

The matrix element $\mathcal{H}^\chi_{T2}$ can be expressed in the same form of Eqs. (36,37) with the operators in the densities of Eqs. (36,37) replaced as descibed in the Appendix.

At this point we want to mention that the densities $\mathcal{A}_T(r), \cdots, \mathcal{J}_T(r)$ are similar to those for the axial-vector form factors $\mathcal{A}(r), \cdots, \mathcal{J}(r)$ [27]. The only difference comes from the γ_0 (γ^4 in Euclidean space) in Eqs. (8,9). This results in the fact that the reduced matrix elements of the operators occurring in Eq. (36) are the same for both tensor and axial-vector densities. The difference in the complete densities is therefore a minus sign in the lower Lorentz structure. The factor γ_4, however, makes the densities for the tensor charges totally different from the axial-vector ones. The effective chiral action expressed in Eq. (18) contains in principle all order of the effective chiral Lagrangians. Its imaginary part generates the well-known Wess-Zumino-Witten Lagrangian [22, 51]. In order to produce this Lagrangian correctly, one should not regularize the imaginary part if the momentum-dependence of the constituent quark mass is turned off. Thus, the contributions from the imaginary part of the action to an observable do not have any regularization. As for the tensor charges, it is the other way around, that is, the real part contains no regularization but the imaginary part. This is due to the presence of the factor γ_4 in the tensor operator that switches the real and imaginary parts.

V. AXIAL-VECTOR FORM FACTORS IN THE CHIRAL QUARK-SOLITON MODEL

In this Section we will discuss shortly the axial-vector form factors $G^\chi_A(Q^2)$ calculated in the χQSM. The general baryon matrix element is decomposed into its Lorentz-structure as given below:

$$\langle N_{s'}(p')|\overline{\psi}(0)\gamma^5\gamma^\mu\lambda^\chi\psi(0)|N_s(p)\rangle = u_{s'}(p')\left[G^\chi_A(Q^2)\gamma^\mu + G^\chi_P(Q^2)\frac{q^\mu}{2M} + G^\chi_T(Q^2)\frac{n^\mu}{2M}\right]\gamma^5 u_s(p)\,. \tag{40}$$

Being similar to the evaluation of the $H_T^\chi(Q^2)$ form factors discussed in the previous Section, the axial-vector form factors $G_A^\chi(Q^2)$ are obtained in the same framework with the corresponding expressions as:

$$G_A^\chi(Q^2) \;=\; \frac{M}{E}\;\int dr\, r^2\left[j_0(|\boldsymbol{Q}|r)\,\mathcal{G}_0^\chi(r)\;-\;\frac{1}{\sqrt{2}}j_2(|\boldsymbol{Q}|r)\,\mathcal{G}_2^\chi(r)\right]. \tag{41}$$

The nucleon matrix element $\mathcal{G}_0^\chi(r)$ is given in the SU(3) case as:

$$\begin{aligned}
\mathcal{G}_0^{\chi=3,8}(r) \;=\; & -\sqrt{\frac{1}{3}}\langle D_{\chi 3}^{(8)}\rangle_N\mathcal{A}_0(r)\;+\;\frac{1}{3\sqrt{3}}\frac{1}{I_1}\langle D_{\chi 8}^{(8)}J_3\rangle_N\mathcal{B}_0(r)\\
& -\sqrt{\frac{1}{3}}\frac{1}{I_2}\langle d_{ab3}D_{\chi a}^{(8)}J_b\rangle_N\mathcal{C}_0(r)\;-\;\frac{1}{3\sqrt{2}}\frac{1}{I_1}\langle D_{\chi 3}^{(8)}\rangle_N\mathcal{D}_0(r)\\
& -\frac{2}{3\sqrt{3}}\frac{K_1}{I_1}M_8\langle D_{83}^{(8)}D_{\chi 8}^{(8)}\rangle_N\mathcal{B}_0(r)+\frac{2}{\sqrt{3}}\frac{K_2}{I_2}M_8\langle d_{ab3}D_{8a}^{(8)}D_{\chi b}^{(8)}\rangle_N\mathcal{C}_0(r)\\
& -\sqrt{\frac{1}{3}}\left[2M_1\langle D_{\chi 3}^{(8)}\rangle_N+\frac{2}{\sqrt{3}}M_8\langle D_{88}^{(8)}D_{\chi 3}^{(8)}\rangle_N\right]\mathcal{H}_0(r)\\
& +\frac{2}{3\sqrt{3}}M_8\langle D_{83}^{(8)}D_{\chi 8}^{(8)}\rangle_N\mathcal{I}_0(r)-\frac{2}{\sqrt{3}}M_8\langle d_{ab3}D_{\chi a}^{(8)}D_{8b}^{(8)}\rangle_N\mathcal{J}_0(r)\,,
\end{aligned} \tag{42}$$

$$\mathcal{G}_0^0(r) \;=\; \frac{1}{3}\frac{1}{I_1}\langle J_3\rangle_N\mathcal{B}_0(r)-\frac{2}{3}\frac{K_1}{I_1}M_8\langle D_{83}^{(8)}\rangle_N\mathcal{B}_0(r)+\frac{2}{3}M_8\langle D_{83}^{(8)}\rangle_N\mathcal{I}_0(r)\,, \tag{43}$$

where the densities $\mathcal{A}_0,\cdots,\mathcal{J}_0$ are related to those of the tensor form factor by simply dropping the γ_4 appearing in the densities $\mathcal{A}_{T0},\cdots,\mathcal{J}_{T0}$ given in the Appendix.

The axial-vector form factors in the SU(2) χQSM is given as

$$\mathcal{G}_0^3(r) \;=\; -\sqrt{\frac{1}{3}}\langle D_{33}\rangle_N\mathcal{A}_0(r)\;-\;\frac{1}{3\sqrt{2}}\frac{1}{I_1}\langle D_{33}\rangle_N\mathcal{D}_{T0}(r) \tag{44}$$

$$\mathcal{G}_0^0(r) \;=\; \frac{1}{3}\frac{1}{I_1}\langle J_3\rangle_N\mathcal{B}_0(r). \tag{45}$$

The above given expressions for the axial-vector form factors are equivalent to those obtained in [27]. However, the given numerical results in the present work are obtained by taking $\lambda^0 = 1$ whereas those of the work [27] correspond to taking $\lambda^0 = \sqrt{2/3}$.

VI. TENSOR AND AXIAL-VECTOR CHARGES

In the case of the tensor and axial-vector charges, i.e. the form factors at $Q^2 = 0$, it is possible to write the corresponding expressions in a very compact way. At the point $Q^2 = 0$ the second terms in Eqs. (35,41) vanish and the spherical Bessel function of the first terms is reduced to unity. Hence, all the model-dependent dynamical parts are just given by the integerals such as $\int drr^2\mathcal{A}$, which are just simple numbers. The residual factors such as $\langle D_{33}^{(8)}\rangle_N$ are SU(3) Clebsch-Gordan coefficients which can be derived by the expression given in the Appendix.

The expressions Eqs. (35,41) for the tensor and axial-vector charges can be reduced to the following expressions in SU(3):

$$g^{\chi=3,8} \;=\; -\frac{\langle D_{\chi 3}^{(8)}\rangle_N}{\sqrt{3}}\,A\;+\;\frac{\langle D_{\chi 8}^{(8)}J_3\rangle_N}{3I_1\sqrt{3}}\,B\;-\;\frac{\langle d_{ab3}D_{\chi a}^{(8)}J_b\rangle_N}{I_2\sqrt{3}}\,C\;-\;\frac{\langle D_{\chi 3}^{(8)}\rangle_N}{3I_1\sqrt{2}}\,D$$

$$
\begin{aligned}
&- M_8\frac{2K_1\langle D^{(8)}_{83}D^{(8)}_{\chi 8}\rangle_N}{3I_1\sqrt{3}}B + M_8\frac{2K_2\langle d_{ab3}D^{(8)}_{8a}D^{(8)}_{\chi b}\rangle_N}{3I_2}\,C \\
&-\sqrt{\frac{1}{3}}\left[2M_1\langle D^{(8)}_{\chi 3}\rangle_N + \frac{2}{\sqrt{3}}M_8\langle D^{(8)}_{88}D^{(8)}_{\chi 3}\rangle_N\right]\,H \\
&+M_8\frac{2\langle D^{(8)}_{83}D^{(8)}_{\chi 8}\rangle_N}{3\sqrt{3}}\,I - M_8\frac{2\langle d_{ab3}D^{(8)}_{\chi a}D^{(8)}_{8b}\rangle_N}{\sqrt{3}}\,J\,,
\end{aligned} \tag{46}
$$

$$
g^0 \;=\; \frac{\langle J_3\rangle_N}{3I_1}\,B - M_8\frac{2K_1\langle D^{(8)}_{83}\rangle_N}{3I_1}\,B + M_8\frac{2\langle D^{(8)}_{83}\rangle_N}{3}\,I\,, \tag{47}
$$

and those in SU(2):

$$
g^3 \;=\; -\frac{\langle D_{33}\rangle_N}{\sqrt{3}}A \;-\; \frac{\langle D_{33}\rangle_N}{3I_1\sqrt{2}}D \tag{48}
$$

$$
g^0 \;=\; \frac{\langle J_3\rangle_N}{3I_1}B. \tag{49}
$$

We list in Table VI in the Appendix the values for the densities $\mathcal{A}_{T0},\cdots,\mathcal{J}_{T0}$ and $\mathcal{A}_0,\cdots,\mathcal{J}_0$ integrated over r with the weight r^2as obtained in the χQSM. In the case of SU(3) symmetry, i.e. without m_s corrections, the nucleon matrix elements as used in Eqs. (46,47, 48,49) are given in Table V in the Appendix. All results at $Q^2 = 0$ as given in the present work can be reproduced by using the results listed in Tables VI,V together with Eqs. (46,47, 48,49).

VII. RESULTS AND DISCUSSION

We are now in a position to discuss the results for the tensor form factors $H_T(Q^2)$ of Eq. (1). We want to mention that all model parameters are the same as in the former works [26–29, 43–46]. For a given M, the regularization cut-off parameter Λ and the current quark mass $\overline{m}$ in the Lagrangian are then fixed to the pion decay constant f_π and the pion mass m_π, respectively. Throughout this work the strange current quark mass is fixed to $m_{\rm s} = 180\,\mathrm{MeV}$ which approximately reproduces the kaon mass. Hence, we do not have any further free parameter for the present investigation. Especially, this is the merit of the χQSM which enables us to investigate all baryon form factors within exactly the same framework. In the present case, these are the tensor and axial-vector form factors. We also apply the symmetry-conserving quantization as found in Ref. [33]. The experimental proton electric charge radius is best reproduced in the χQSM with the constituent quark mass $M = 420$ MeV which is thus our preferred value. Nevertheless we have checked in this work that the results for the tensor form factors are rather stable with M varied, so that we present all results with $M = 420$ MeV.

We first concentrate on the tensor charges $g_T^\chi = H_T^\chi(0)$ for the singlet and non-singlet components corresponding respectively to $\chi = 0$ and $\chi = 3, 8$. In Table I we list the results of the tensor charges for the singlet and nonsinglet components, comparing them with those of the axial-vector charges. In Ref. [30] the SU(2) isovector and isosinglet tensor charges were obtained to be $g_T^3 = 1.45$ and $g_T^0 = 0.69$ which are in agreement with those obtained in the present work. From the above results, we find the following interesting inequalities

$$
g_T^\chi > g_A^\chi, \tag{50}
$$

Table I: Tensor charges in comparison with the axial-vector charges. Both charges, in SU(2) and SU(3), have been calculated with the same set of parameters in the present work. The results for the non-relativistic quark model are also given.

	g_T^0	g_T^3	g_T^8	g_A^0	g_A^3	g_A^8	Δu	δu	Δd	δd	Δs	δs
χQSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
χQSM SU(2)	0.75	1.44	$--$	0.45	1.21	$--$	0.82	1.08	-0.37	-0.32	$--$	$--$
NRQM	1	5/3	$--$	1	5/3	$--$	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$--$	$--$

that is, the tensor charges turn out to be in general larger than the axial-vector charges. These inequalities are also true for the SU(2) results. As mentioned already, the only difference between the tensor and axial-vector operators is due to the factor of γ_4. Therefore, both charges coincide in the non-relativistic limit [17, 52]. As discussed already in Ref. [30], this can be qualitatively understood from the asymptotics of both charges in soliton size R_0. The tensor charges show generally weaker dependence on the soliton size than the axial-vector ones do [30]:

$$\begin{array}{ll} g_A^3 \sim (MR_0)^2 & g_T^3 \sim MR_0 \\ g_A^0 \sim \dfrac{1}{(MR_0)^4} & g_T^0 \sim \dfrac{1}{MR_0} \end{array} . \tag{51}$$

As a result, the tensor charges from the χQSM turn out to be closer to those from the nonrelativistic quark model (NRQM) (the limit of the small soliton size $R_0 \to 0$) than the corresponding axial-vector charges. A similar conclusion was drawn in the bag model [52].

The tensor charges were studied independently within the SU(2) χQSM in Ref. [37, 54] in which the nucleon structure functions have been calculated. The tensor and axial-vector charges were derived as the first moments of the longitudinally and transversely polarized distribution functions, respectively. Reference [37] obtained the SU(2) axial-vector charges as $g_A^0 = 0.35$ and $g_A^3 = 1.41$, and the tensor charges as $g_T^0 = 0.56$ and $g_T^3 = 1.22$ whereas Ref. [54] used $g_A^0 = 0.35$, $g_A^3 = 1.31$, $g_T^0 = 0.68$ and $g_T^3 = 1.21$. While the values for the singlet from Ref. [37, 54] are similar to those of the present work, the values for g_A^3 and g_T^3 seem to be somewhat different. Moreover, their results of g_A^3 and g_T^3 do not show the inequality of Eq. (50). However, note that their ratios of $g_A^0/g_A^3 \simeq 0.25(0.27)$ and $g_T^0/g_T^3 \simeq 0.46(0.56)$ and the SU(2) ratios of the present work $g_A^0/g_A^3 = 0.37$ and $g_T^0/g_T^3 = 0.52$ show the same deviation from the non-relativiestic quark model value $g^0/g^3 = 0.6$.

In the case of the SU(3) χQSM, the tensor charges were already studied in [32]. However, the former calculation was done without the symmetry-conserving quantization which ensures the correct realization of the Gell-Mann-Nishijima formula, and yielded the following results: $g_T^0 = 0.70$, $g_T^3 = 1.54$, and $g_T^8 = 0.42$. Compared to the present results listed in Table I, the previous results are deviated from the present ones by about $6 \sim 10\,\%$.

The singlet and nonsinglet tensor charges can be decomposed into the tensor charges for each flavor as follows:

$$\delta u = \frac{1}{2}\left(\frac{2}{3}g_T^0 + g_T^3 + \frac{1}{\sqrt{3}}g_T^8\right) ,$$
$$\delta d = \frac{1}{2}\left(\frac{2}{3}g_T^0 - g_T^3 + \frac{1}{\sqrt{3}}g_T^8\right) ,$$

$$\delta s = \frac{1}{3}\left(g_T^0 - \sqrt{3}g_T^8\right) . \tag{52}$$

In Table I, we list the results for the flavor-decomposed tensor charges of the nucleon in comparison with the corresponding axial-vector ones. As in the case of the axial-vector charges, the rotational $1/N_c$ corrections are also crucial for the tensor charges. On the other hand, the m_s corrections that come from both the operators and wave function corrections turn out to be rather small, i.e. below 5 %. Moreover, the Dirac-sea quark contribution to the form factor $H_T(Q^2)$ is almost negligible. The strange tensor charge turns out to be tiny. Compared to the work [54], Table I shows the same tendency, namely: $\delta u > \Delta u$ and $\delta d > \Delta d$.

Table II: The scale independent quantity $|\delta d/\delta u|$. References [30], [54], and [35] correspond respectively to the the SU(2) χQSM, the same model by Wakamatsu, and the SU(3) infinite momentum frame χQSM. The values of the works [17–19, 55] were obtained in the SU(6) symmetric CQM. The SU(6) symmetric Ansatz induces a ratio of 1/4. Reference [16] represents a global analysis of SIDIS experimental data.

Proton	This work	SU(2)[30]	Ref. [54]	IMF[35]	CQM[17–19, 55]	Lattice[21]	SIDIS[16]	NR
$\lvert\delta d/\delta u\rvert$	0.30	0.36	0.28	0.27	0.25	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

Table II lists the ratio of the tensor charges $|\delta d/\delta u|$ for the proton, compared to different approaches. Note that this ratio is independent of any renormalization scale [53]. For comparision, we first consider the results of the SU(2) χQSM [30] as well as those of the same model by Ref. [54]. We also compare the present results with those of the SU(3) infinite-momentum frame χQSM [35] and of the following SU(6)-symmetric model: MIT-bag model [17], the harmonic oscillator (HO) and hypercentral (HYP) light-cone quark models of [18, 19]. Furthermore, Ref. [55] followed the approach of Ref. [19] by using a proton wave function derived in a quark model [56]. The results of the CQM [18, 19, 55] correspond to the three valence quark contribution without the Dirac sea in the framework of the light-cone quantization. The approximations used in the present work and in the SU(3) infinite momentum framework χQSM of Ref. [35] are quite opposite each other. In Ref. [35] the rotation of the χQSM soliton can be taken exactly while the Dirac-sea contribution is truncated. In the present formalism the whole Dirac-sea contribution is included while the rotation of the soliton is taken perturbatively. In the case of the SU(6)-symmetric models [17–19, 55], the SU(6)-symmetric Ansatz induces a ratio $|\delta d/\delta u| = 1/4$. As for the experimental value of $|\delta d/\delta u|$ we take the results of Ref. [16], where $\delta u = 0.54^{+0.09}_{-0.22}$ and $\delta d = -0.23^{+0.09}_{-0.16}$ at $\mu^2 = 0.8\,\mathrm{GeV}^2$ were obtained. Compared to that value, all theoretical results look underestimated, however, still within uncertainty.

From the results of the SU(2) and SU(3) χQSM listed in Table II, we find that the SU(2) result from Ref. [30] seems deviated from that of Ref. [54] and also from that of the present work. The difference of the present work to Ref. [30] is mainly due to the fact that the g_T^0 differ by a value of 0.06, i.e. 8%. This difference could be explained by the fact that the soliton profile and discretization parameters used in Ref. [30] are different from the present work. Taking into account this, a deviation of 8% is an acceptable one. In comparisson to the work Ref. [54] the ratio $\delta d/\delta u$ is comparable to the present one since both the δd and δu are approximately by the same factor smaller as compared to the results of the present work.

In Ref. [19], the following relation was presented:

$$2\delta u \;=\; \Delta u + \frac{4}{3}, \quad 2\delta d \;=\; \Delta d - \frac{1}{3}\,, \tag{53}$$

which is compatible with the Soffer inequality [57]. It is worthwhile to note that this relation is numerically approximately fulfilled by the results of the present work.

In the following, we will compare our results to the lattice calculation Ref. [21]. The lattice results for the tensor form factors were derived at a renormalization scale of $\mu^2 = 4\,\mathrm{GeV}^2$ and are linearly extrapolated to the physical pion mass as well as to the continuum. Disconnected quark-loop diagrams were not considered. In the χQSM, the renormalization scale is given by the cut-off mass of the regularization, which is approximately $\mu^2 \approx 0.36\,\mathrm{GeV}^2$. This value is related implicitly to the size of the QCD instantons $\overline{\rho} \approx 0.35\,\mathrm{fm}$ [24, 25]. We use Eq. (15) in order to evolve the lattice and SIDIS results [16] from $4\,\mathrm{GeV}^2$ and $0.80\,\mathrm{GeV}^2$, respectively, to the renormalization scale of the χQSM $0.36\,\mathrm{GeV}^2$. We compare the present results with those of the lattice calculations as follows:

$$\begin{array}{rll}
\text{SIDIS [16] } (0.80\,\mathrm{GeV}^2): & \delta u = 0.54^{+0.09}_{-0.22}\,, & \delta d = -0.231^{+0.09}_{-0.16}, \\
\text{SIDIS [16] } (0.36\,\mathrm{GeV}^2): & \delta u = 0.60^{+0.10}_{-0.24}\,, & \delta d = -0.26^{+0.1}_{-0.18}, \\
\text{Lattice [21] } (4.00\,\mathrm{GeV}^2): & \delta u = 0.86 \pm 0.13\,, & \delta d = -0.21 \pm 0.005\,, \\
\text{Lattice [21] } (0.36\,\mathrm{GeV}^2): & \delta u = 1.05 \pm 0.16\,, & \delta d = -0.26 \pm 0.01\,, \\
\chi\text{QSM } (0.36\,\mathrm{GeV}^2): & \delta u = 1.08\,, & \delta d = -0.32\,,
\end{array}$$

from which we find that the results are generally in a good agreement with those of the lattice calculation [21]. However, both approaches disagree with the SIDIS results [16] for δu by nearly a factor of 2.

We now turn to the tensor form factors $H_T^\chi(Q^2)$ calculated up to the momentum transfer of $Q^2 \leq 1\,\mathrm{GeV}^2$. The tensor form factors $H_T^\chi(Q^2)$ expressed in Eq. (35) consists of two densities. While only the first one determines the tensor charges at $Q^2 = 0$, both densities are responsible for $H_T^\chi(Q^2)$. In the upper left panel of Fig. 1, the up and down tensor form factors are shown together with the corresponding axial-vector form factors. Interestingly, we find that the general behaviors of these form factors are very similar. In particular, δd and Δd look pretty much similar to each other. In the upper right panel, the strange tensor and axial-vector form factors are drawn. In contrast to the nonstrange form factors, the strange tensor form factor seems to be very different from the axial-vector one. The Q^2 dependence of the strange tensor form factor is somewhat peculiar. It behaves like a neutral form factor. Moreover, the strange tensor charge turns out to be very small, compared to the axial-vector one. It implies that the tensor form factors are "less relativistic".

In the lattice work [21] the tensor form factors $\delta u(Q^2)$ and $\delta d(Q^2)$ were calculated up to a momentum transfer of $Q^2 = 3.5\,\mathrm{GeV}^2$. In the lower panel of Fig. 1, we compare our scaled form factors $\delta q(Q^2)/\delta q(0)$ with those of the lattice calculation [21]. In fact, these scaled form factors are independent of the renormalization scale. While the lattice result for δu decreases almost linearly as Q^2 increases, that of the present work falls off more rapidly. Thus, the value of δu at $Q^2 = 1\,\mathrm{GeV}^2$ is almost 2.5 times smaller than that of the lattice calculation. Note, however, that this is an expected result. A similar behavior was also obtained for the $\Delta(1232)$ electric quadrupole form factor as shown in Ref. [58]. These differences of the Q^2 dependence can be understood from the fact that lattice calculations tend to yield rather flat form factors because of the heavy pion mass employed. This has been shown explicitly for the nucleon isovector form factor $F_1^{p-n}(Q^2)$ on the lattice [59].

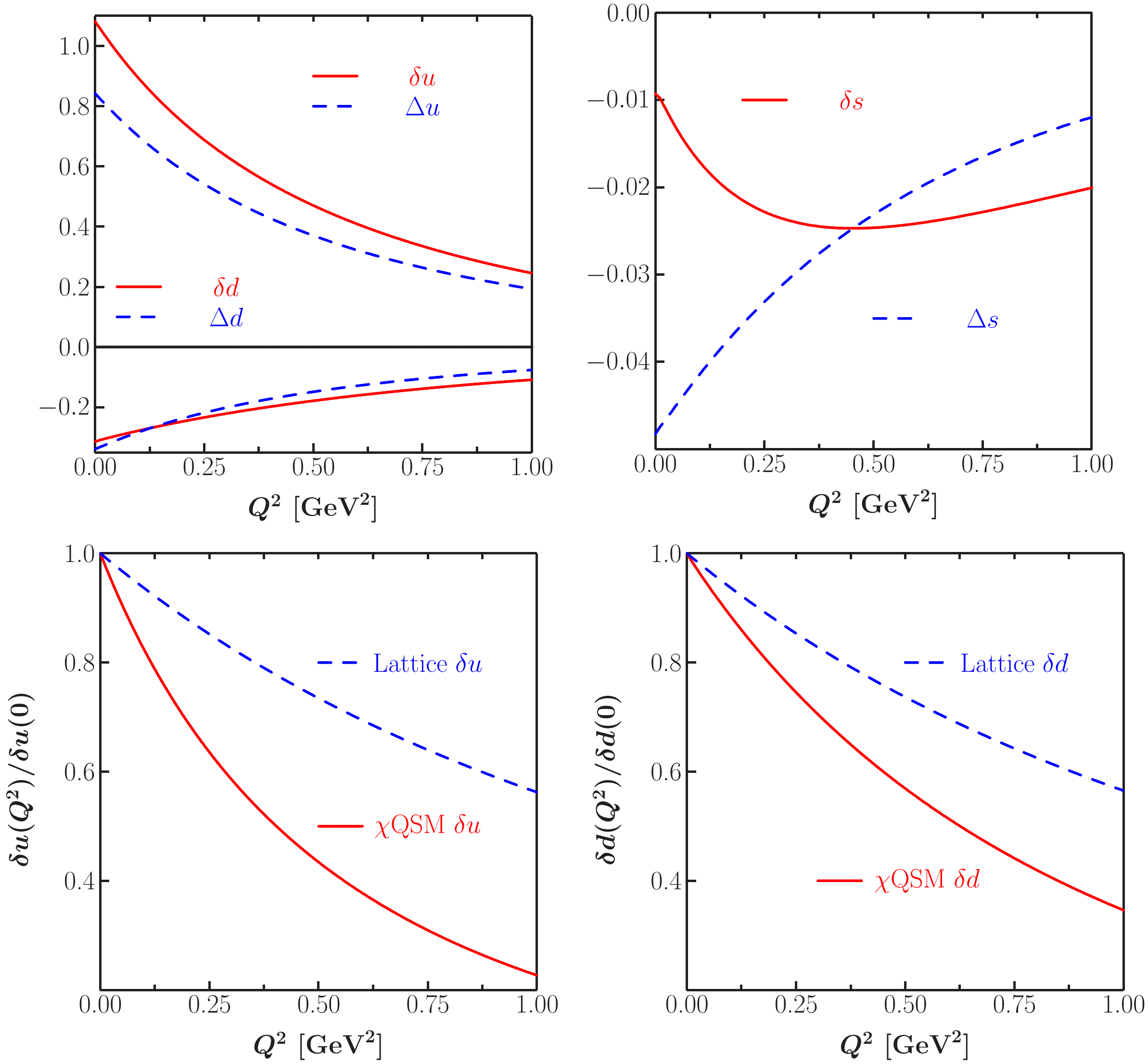

Figure 1: (Color online) flavor tensor form factors $\delta u(Q^2)$, $\delta d(Q^2)$ and $\delta s(Q^2)$ for the proton. In the upper panel, we compare the tensor form factors with the axial-vector ones. The red (solid) curves show the tensor form factors whereas the blue (dashed) ones represent the axial-vector form factors. In the lower panel, we compare the present results of the renormalization-independent scaled tensor form factors $\delta u(Q^2)/\delta u(0)$, $\delta d(Q^2)/\delta d(0)$ with those of the lattice QCD [21]. The red (solid) curves designate the χQSM form factors of this work while the blue (dashed) ones corrrespond to the factors from the lattice QCD calculation.

In general, the form factors from the χQSM are well reproduced by the dipole formula

$$H_T(Q^2) = \frac{H_T(0)}{(1+Q^2/M_{\mathrm{d}}^2)^2} \tag{54}$$

with the dipole mass M_{d}. A direct fit leads to the dipole masses corresponding to the tensor form factors for $\chi = 0, 3, 8$ and the up and down form factors $\delta u(Q^2)$ and $\delta d(Q^2)$ as listed

in Table III. Note that, however, the strange tensor form factors cannot be fitted in terms of the dipole type.

Table III: Dipole masses $M_{\rm d}$ for the tensor form factors $H_T^\chi(Q^2)$, $\delta u(Q^2)$ and $\delta d(Q^2)$. Given are the values of the tensor form factors for each flavor at $Q^2 = 0$ and the dipole masses in units of GeV, which reproduce the present results.

Proton	$H_T^0(Q^2)$	$H_T^3(Q^2)$	$H_T^8(Q^2)$	$\delta u(Q^2)$	$\delta d(Q^2)$
$Q^2 = 0$	0.76	1.40	0.45	1.08	-0.32
$M_{\rm d}$	0.851	1.03	0.984	0.980	1.24

Table IV: Tensor charges for $\delta q(0)$ for the baryon octet.

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
δu	1.08	-0.32	-0.03	1.08	0.53	-0.02	-0.32	-0.02
δd	-0.32	1.08	-0.03	-0.02	0.53	1.08	-0.02	-0.32
δs	-0.01	-0.01	0.79	-0.29	-0.29	-0.29	1.06	1.06

For completeness, we list in Table IV the tensor charges δq for the baryon octet. Having scrutinized the results in Table IV, we find the following relations:

$$\begin{aligned}
\delta u_p &= \delta d_n, & \delta u_n &= \delta d_p, & \delta u_\Lambda &= \delta d_\Lambda, & \delta u_{\Sigma^+} &= \delta d_{\Sigma^-}, \\
\delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0}, \\
\delta s_p &= \delta s_n, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-},
\end{aligned} \tag{55}$$

which are the consequence of the assumed isospin symmetry in the present work. Generally, in $SU(3)$ flavor symmetry we have the following relations:

$$\begin{aligned}
\delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-}, \\
\delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.
\end{aligned} \tag{56}$$

By comparing the above relations with the numbers given in Table IV we can see the overall smallness of SU(3) symmetry breaking contributions for the tensor charges of all octet baryons. Similar relations to Eqs. (55, 56) can be found also for the axial-vector charges and magnetic moments of the octet baryon [60, 61].

VIII. SUMMARY AND CONCLUSION

In the present work we investigated the tensor form factors $H_T(Q^2)$ of the SU(3) baryons, which are deeply related to the chiral-odd generalized parton distribution $H_T(x, \xi, t)$. We used the SU(3) self-consistent chiral quark-soliton model (χQSM) with symmetry-conserving quantization in order to calculate the tensor charges and form factors up to the momentum transfer $Q^2 \leq 1\,\mathrm{GeV}^2$, taking into account linear rotational $1/N_c$ corrections and linear $m_{\rm s}$

corrections. All parameters of the model including the constituent quark mass have been already fixed in reproducing the meson and nucleon properties. No additional parameter has been fitted in the present calculation.

We first computed the flavor singlet and nonsinglet tensor charges of the nucleon: $g_T^0 = 0.76$, $g_T^3 = 1.40$, and $g_T^8 = 0.45$. As for the flavor-decomposed tensor charges $\delta q = H_T^q(0)$, we obtained the following results: $\delta u = 1.08$, $\delta d = -0.32$ and $\delta s = -0.01$. We found that for these tensor charges the Dirac-sea contribution as well as the effects of flavor SU(3) symmetry breaking are negligibly small. We compared the present results of the tensor form factors $H_T^{u,d}(Q^2)$ with those of the lattice QCD [21]. For the up and down tensor charges, i.e. $H_T^{u,d}(0)$, the results are in good agreement with the lattice data. However, the present results of the tensor form factors fall off faster than those from the lattice QCD, as Q^2 increases. The reason for this lies in the fact that the heavier pion mass utilized in the lattice calculation causes generally flat form factors. We also presented the tensor charges of the baryon octet. The results indicated that the effects of SU(3) symmetry breaking turn out to be negligibly small.

The second and third tensor form factors, i.e. $E_T(Q^2)$ and $\tilde{H}_T(Q^2)$, will be discussed elsewhere. The corresponding investigation is under way.

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Appendix A: Densities

In this Appendix, we provide the densities for the tensor form factors given in Eq. (36) which comprise $\mathcal{A}_{T0}(r), ..., \mathcal{J}_{T0}(r)$ and $\mathcal{A}_{T2}(r), ..., \mathcal{J}_{T2}(r)$. The corresponding vector operators O_1 in the spherical tensor operator notation of Ref. [62] for the individual densites are given as:

for $\mathcal{A}_{T0}(r), ..., \mathcal{J}_{T0}(r) \;\rightarrow\; O_1 = \sigma_1$
for $\mathcal{A}_{T2}(r), ..., \mathcal{J}_{T2}(r) \;\rightarrow\; O_1 = \sqrt{4\pi}\;\{Y_2 \otimes \sigma_1\}_1$

In the following, the sums run freely over all single-quark levels including valence ones $|v\rangle$ except that the sum over n_0 is constrained to negative-energy levels:

$$\frac{1}{N_c}\mathcal{A}_T(r) \;=\; \langle v||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||v\rangle - \frac{1}{2}\sum_n \mathrm{sign}(\varepsilon_n)\sqrt{2G_n+1}\langle n||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n\rangle$$
$$\frac{1}{N_c}\mathcal{B}_T(r) \;=\; \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_v-\varepsilon_n}(-)^{G_n}\langle v||r\rangle\gamma_4 O_1\langle r||n\rangle\langle n||\tau_1||v\rangle$$

$$
\begin{aligned}
& -\frac{1}{2}\sum_{n,m}\mathcal{R}_3(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\tau_1||m\rangle\langle m||r\rangle\gamma_4 O_1\langle r||n\rangle \\
\frac{1}{N_c}\mathcal{T}_T(r) &= \sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_v-\varepsilon_{n^0}}\langle v||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n^0\rangle\langle n^0\mid v\rangle \\
& -\sum_{n,m}\mathcal{R}_3(\varepsilon_n,\varepsilon_{m^0})\sqrt{2G_n+1}\langle m^0||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n\rangle\langle n\mid m^0\rangle \\
\frac{1}{N_c}\mathcal{D}_T(r) &= \sum_{\varepsilon_n}\frac{\mathrm{sign}(\varepsilon_n)}{\varepsilon_v-\varepsilon_n}(-)^{G_n}\langle v||\tau_1||n\rangle\langle n||r\rangle\gamma_4\{O_1\otimes\tau_1\}_1\langle r||v\rangle \\
& +\frac{1}{2}\sum_{n,m}\mathcal{R}_6(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\tau_1||m\rangle\langle m||r\rangle\gamma_4\{O_1\otimes\tau_1\}_1\langle r||n\rangle \\
\frac{1}{N_c}\mathcal{H}_T(r) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_v-\varepsilon_n}\langle v||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r|n\rangle\langle n|\gamma^0|v\rangle \\
& -\frac{1}{2}\sum_{n,m}\mathcal{R}_5(\varepsilon_n,\varepsilon_m)\sqrt{2G_m+1}\langle m||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n\rangle\langle n|\gamma^0|m\rangle \\
\frac{1}{N_c}\mathcal{I}_T(r) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_v-\varepsilon_n}(-)^{G_n}\langle v||r\rangle\gamma_4 O_1\langle r||n\rangle\langle n|\gamma^0\tau_1||v\rangle \\
& -\frac{1}{2}\sum_{n,m}\mathcal{R}_5(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\gamma^0\tau_1||m\rangle\langle m||r\rangle\gamma_4 O_1\langle r||n\rangle \\
\frac{1}{N_c}\mathcal{J}_T(r) &= \sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_v-\varepsilon_{n^0}}\langle v||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n^0\rangle\langle n^0|\gamma^0|v\rangle \\
& -\sum_{n,m}\mathcal{R}_5(\varepsilon_n,\varepsilon_{m^0})\sqrt{2G_m+1}\langle m^0||r\rangle\gamma_4\{O_1\otimes\tau_1\}_0\langle r||n\rangle\langle n|\gamma^0|m^0\rangle\,. \qquad \text{(A1)}
\end{aligned}
$$

where we take the notation for the reduced matrix elements of the r-dependent states as schematically given below:

$$
\begin{aligned}
\langle n||r\rangle O_1\langle r||m\rangle &= \left(\, A(r)\langle i_n|| \;\; B(r)\langle j_n||\,\right) O_1 \begin{pmatrix} C(r)||i_m\rangle \\ D(r)||j_m\rangle \end{pmatrix} \\
&= A(r)C(r)\langle i_n||O_1||i_m\rangle + B(r)D(r)\langle j_n||O_1||j_m\rangle.
\end{aligned}
$$

The functions $A(r)$, $B(r)$, $C(r)$, $D(r)$ and grand-spin states i_n, i_m, j_n, j_m can be found in Ref. [63].

The regularization functions $\mathcal{R}_i(\varepsilon_n,\varepsilon_m)$ appearing in Eq.(A1) are given by

$$
\begin{aligned}
\mathcal{R}_3(\varepsilon_n,\varepsilon_m) &= \frac{1}{2\sqrt{\pi}}\int_{1/\Lambda^2}^{\infty}\frac{du}{\sqrt{u}}\Big[\frac{1}{u}\frac{e^{-\varepsilon_n^2 u}-e^{-\varepsilon_m^2 u}}{\varepsilon_m^2-\varepsilon_n^2}-\frac{\varepsilon_n e^{-u\varepsilon_n^2}+\varepsilon_m e^{-u\varepsilon_m^2}}{\varepsilon_m+\varepsilon_n}\Big], \\
\mathcal{R}_5(\varepsilon_n,\varepsilon_m) &= \frac{1}{2}\frac{\mathrm{sign}\varepsilon_n-\mathrm{sign}\varepsilon_m}{\varepsilon_n-\varepsilon_m}, \\
\mathcal{R}_6(\varepsilon_n,\varepsilon_m) &= \frac{1}{2}\frac{1-\mathrm{sign}(\varepsilon_n)\mathrm{sign}(\varepsilon_m)}{\varepsilon_n-\varepsilon_m}.
\end{aligned}
$$

Appendix B: Baryon matrix elements and integrated densities

All appearing baryon matrix elements in this work are calculated by using the following relation:

$$\langle B'(\mathcal{R}')|D^{(n)}_{\chi\alpha}|B(\mathcal{R})\rangle = \sqrt{\frac{\dim(\mathcal{R}')}{\dim(\mathcal{R})}}(-1)^{\frac{1}{2}Y'_s+S'_3}(-1)^{\frac{1}{2}Y_s+S_3} \tag{B1}$$

$$\sum_\gamma \begin{pmatrix} \mathcal{R}' & n & \mathcal{R}_\gamma \\ Q'_y & \chi & Q_y \end{pmatrix} \begin{pmatrix} \mathcal{R}' & n & \mathcal{R}_\gamma \\ \overline{Q}_{s'} & \alpha & \overline{Q}_s \end{pmatrix}, \tag{B2}$$

where $B(\mathcal{R})$ represents a baryon from the SU(3) representation R with the flavor quantum numbers $Q_y = YII_3$ and spin quantum numbers $Q_s = Y_sSS_3$ ($\overline{Q_s} = -Y_sS - S_3$). The quantities in brackets represent the SU(3) Clebsch-Gordan coefficients.

Table V: SU(3) Nucleon matrix elements

$\langle D^{(8)}_{33}\rangle_N$	$\langle D^{(8)}_{83}\rangle_N$	$\langle D^{(8)}_{38}\rangle_N$	$\langle D^{(8)}_{88}\rangle_N$	$\langle d_{ab3}D^{(8)}_{3a}J_b\rangle_N$	$\langle d_{ab3}D^{(8)}_{8a}J_b\rangle_N$
$-I_3\frac{7}{15}$	$-\frac{\sqrt{3}}{30}$	$I_3\frac{\sqrt{3}}{15}$	$\frac{3}{10}$	$I_3\frac{7}{30}$	$\frac{\sqrt{3}}{60}$

Table VI: Integrated densities for tensor and axial-vector charges with the constituent quark mass $M = 420$ MeV and numerical parameters fixed as described in the text. We use here the following notations: $A = \int drr^2\mathcal{A}(r)$, $B = \int drr^2\mathcal{B}(r)$, $C = \int drr^2\mathcal{C}(r)$, $D = \int drr^2\mathcal{D}(r)$, $H = \int drr^2\mathcal{H}(r)$, $I = \int drr^2\mathcal{I}(r)$, $J = \int drr^2\mathcal{J}(r)$

	A	B	C	D	H	I	J
Tensor	5.22	4.75	-2.46	5.84	-0.02	2.43	-1.66
Axial-Vector	4.20	2.86	-2.10	5.46	0.17	1.20	-1.33

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